

Confidence Intervals One Sample Proportion Free Response

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Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

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Question 1

Qualification: AP Statistics

Areas: Confidence Intervals, Probability

Subtopics: Interpreting a Confidence Interval, Scatterplot, Justifying Independence, One Sample Z Interval For A Proportion, Combining Random Variables, Correlation, Normal Distribution - Two distributions (combining variables)

Paper: Part-B / Series: 2000 / Difficulty: Somewhat Challenging / Question Number: 6

6. A random sample of 400 married couples was selected from a large population of married couples.

- Heights of married men are approximately normally distributed with mean 70 inches and standard deviation 3 inches.
- Heights of married women are approximately normally distributed with mean 65 inches and standard deviation 2.5 inches.
- There were 20 couples in which the wife was taller than her husband, and there were 380 couples in which the wife was shorter than her husband.

(a) Find a 95 percent confidence interval for the proportion of married couples in the population for which the wife is taller than her husband. Interpret your interval in the context of this question.

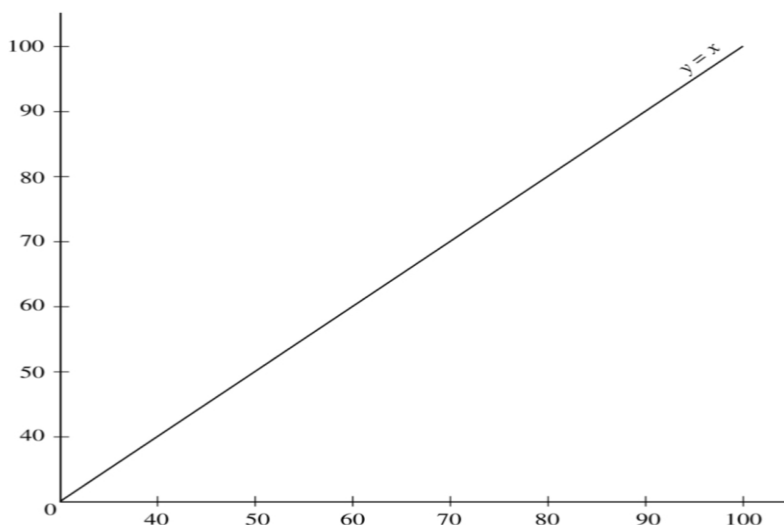
(b) Suppose that a married man is selected at random and a married woman is selected at random. Find the approximate probability that the woman will be taller than the man.

(c) Based on your answers to (a) and (b), are the heights of wives and their husbands independent? Explain your reasoning.

(d) A scatterplot (not shown) of husband's height versus wife's height for the 400 couples in the sample shows an approximately linear relationship with correlation 0.4. On the graph below, sketch an ellipse that could enclose the points on the scatterplot. Be sure to

- label your axes, and
- locate and orient your ellipse correctly with respect to the two axes and the line $y = x$.

Include any information that you think will be helpful in clarifying your sketch.



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Question 2

Qualification: AP Statistics

Areas: Confidence Intervals

Subtopics: Interpreting a Confidence Interval, Constructing a Confidence Interval , One Sample Z Interval For A Proportion, Pooling, Two Sample Z Test For Difference In Proportions

Paper: Part-B / Series: 2002 / Difficulty: Hard / Question Number: 6

6. A survey given to a random sample of students at a university included a question about which of two well-known comedy shows, S or F, students preferred. The students were asked the question, “Do you prefer S or F ?” The responses are shown below.

Preference		
S	F	Total
185	139	324

- (a) Based on the results of this survey, construct and interpret a 95% confidence interval for the proportion of students in the population who would respond S to the question, “Do you prefer S or F ?”
- (b) What is the meaning of “95% confidence” in part (a) ?
- (c) In a follow-up survey, a separate group of randomly selected students was asked “Do you prefer F or S ?” The responses are shown below.

Preference		
S	F	Total
68	88	156

Based on these two surveys, is there evidence that the stated preference depends on the order in which the comedy shows were listed in the survey question? Justify your answer.

- (d) Suppose the test in part (c) indicates that the order in which the shows were listed does make a difference.

Is the pooled value $\frac{185 + 68}{324 + 156} = 0.527$ a reasonable estimate for the proportion of students at the university

who would respond S ? If so, justify your answer. If not, what would be a more reasonable estimate? Explain why.

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Question 3

Qualification: AP Statistics

Areas: Confidence Intervals

Subtopics: Interpreting Confidence Level, Interpreting a Confidence Interval, One Sample Z Interval For A Proportion

Paper: Part-A / Series: 2002-Form-B / Difficulty: Medium / Question Number: 4

4. Each person in a random sample of 1,026 adults in the United States was asked the following question.

“Based on what you know about the Social Security system today, what would you like Congress and the President to do during this next year?”

The response choices and the percentages selecting them are shown below.

Completely overhaul the system	19%
Make some major changes	39%
Make some minor adjustments	30%
Leave the system the way it is now	11%
No opinion	1%

- (a) Find a 95% confidence interval for the proportion of all United States adults who would respond “Make some major changes” to the question. Give an interpretation of the confidence interval and give an interpretation of the confidence level.
- (b) An advocate for leaving the system as it is now commented, “Based on this poll, only 39% of adults in the sample responded that they want some major changes made to the system, while 41% responded that they want only minor changes or no changes at all. Therefore, we should not change the system.” Explain why this statement, while technically correct, is misleading.

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Question 4

Qualification: AP Statistics

Areas: Confidence Intervals

Subtopics: Sample Size, Interpreting a Confidence Interval, Interpreting Confidence Level, Two-Way Tables, One Sample Z Interval For A Proportion

Paper: Part-B / Series: 2003-Form-B / Difficulty: Somewhat Challenging / Question Number: 6

6. Researchers at a large health maintenance organization (HMO) are planning a study of a certain mild illness. They will select a random sample of patients who are ages 35 to 54 and see if they contract the illness in the next year. The researchers are interested in estimating the proportions of men and of women who are likely to develop the illness in each of 4 age-groups: 35-39, 40-44, 45-49, and 50-54.

The researchers plan to include 2,000 patients in the study. Suppose the researchers draw a random sample from all of the patients at this HMO who are ages 35 to 54 and find the following numbers within each gender and age-group.

	Age-Group			
	35-39	40-44	45-49	50-54
Male	350	230	150	60
Female	445	370	245	150

- (a) Suppose that at the end of the study, 10 percent of the females in the 40-44 age-group contracted the illness. Calculate a 95 percent confidence interval to estimate the population proportion of females in this age-group that contracted the illness.

Interpret this confidence interval in the context of this situation.

Interpret the confidence level of 95 percent.

- (b) Suppose that at the end of the study, 10 percent of the males in the 40-44 age-group contracted the illness. The corresponding 95 percent confidence interval to estimate the population proportion of males in this age-group that contracted the illness is (0.061, 0.139).

Note that this interval and the interval in part (a) are of different lengths even though the two sample proportions were identical. What would be an alternative way to allocate a sample of 2,000 subjects so that the 95 percent confidence interval widths for all male age-groups and for all female age-groups (i.e., for all 8 groups) would be the same when the sample proportions are the same? Justify your answer.

- (c) Based on previous studies, researchers believe that the percentages of those who contract the illness will be similar for males and females, and therefore plan to ignore gender when selecting a sample for this study. Previous studies also indicate that the percentages of adults who will contract this illness in the 35-39, 40-44, 45-49, and 50-54 age-groups are anticipated to be 5%, 8%, 20%, and 35%, respectively. How should the sample of 2,000 subjects be allocated with respect to age-groups so that the widths of the 95 percent confidence intervals for the four groups will be approximately the same? Justify your answer.

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Question 5

Qualification: AP Statistics

Areas: Confidence Intervals

Subtopics: Effect of Sampling Without Replacement , Constructing a Confidence Interval , Interpreting a Confidence Interval, One Sample Z Interval For A Proportion

Paper: Part-A / Series: 2010-Form-B / Difficulty: Somewhat Challenging / Question Number: 4

4. A husband and wife, Mike and Lori, share a digital music player that has a feature that randomly selects which song to play. A total of 2,384 songs were loaded onto the player, some by Mike and the rest by Lori. Suppose that when the player was in the random-selection mode, 13 of the first 50 songs selected were songs loaded by Lori.
- (a) Construct and interpret a 90 percent confidence interval for the proportion of songs on the player that were loaded by Lori.
- (b) Mike and Lori are unsure about whether the player samples the songs with replacement or without replacement when the player is in random-selection mode. Explain why this distinction is not important for the construction of the interval in part (a).

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Question 6

Qualification: AP Statistics

Areas: Confidence Intervals, Probability

Subtopics: Tree Diagrams, Constructing a Confidence Interval, Interpreting a Confidence Interval, One Sample Z Interval For A Proportion

Paper: Part-B / Series: 2011 / Difficulty: Somewhat Challenging / Question Number: 6

6. Every year, each student in a nationally representative sample is given tests in various subjects. Recently, a random sample of 9,600 twelfth-grade students from the United States were administered a multiple-choice United States history exam. One of the multiple-choice questions is below. (The correct answer is C.)

In 1935 and 1936 the Supreme Court declared that important parts of the New Deal were unconstitutional. President Roosevelt responded by threatening to

- (A) impeach several Supreme Court justices
- (B) eliminate the Supreme Court
- (C) appoint additional Supreme Court justices who shared his views
- (D) override the Supreme Court's decisions by gaining three-fourths majorities in both houses of Congress

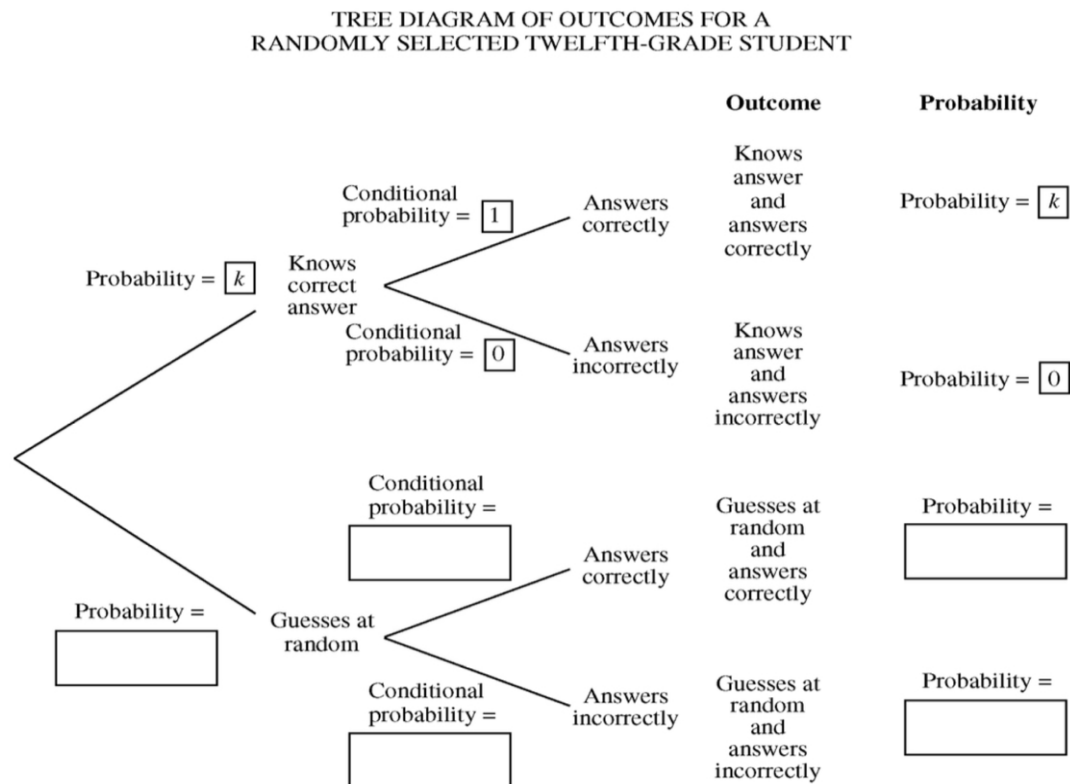
Of the 9,600 students, 28 percent answered the multiple-choice question correctly.

- (a) Let p be the proportion of all United States twelfth-grade students who would answer the question correctly. Construct and interpret a 99 percent confidence interval for p .

Assume that students who actually know the correct answer have a 100 percent chance of answering the question correctly, and students who do not know the correct answer to the question guess completely at random from among the four options.

Let k represent the proportion of all United States twelfth-grade students who actually know the correct answer to the question.

- (b) A tree diagram of the possible outcomes for a randomly selected twelfth-grade student is provided below. Write the correct probability in each of the five empty boxes. Some of the probabilities may be expressions in terms of k .



- (c) Based on the completed tree diagram, express the probability, in terms of k , that a randomly selected twelfth-grade student would correctly answer the history question.

- d) Using your interval from part (a) and your answer to part (c), calculate and interpret a 99 percent confidence interval for k , the proportion of all United States twelfth-grade students who actually know the answer to history question. You may assume that the conditions for inference for the confidence interval have been checked and verified.

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Question 7

Qualification: AP Statistics

Areas: Confidence Intervals

Subtopics: Sample Size, Constructing a Confidence Interval , One Sample Z Interval For A Proportion

Paper: Part-A / Series: 2011-Form-B / Difficulty: Somewhat Challenging / Question Number: 5

5. During a flu vaccine shortage in the United States, it was believed that 45 percent of vaccine-eligible people received flu vaccine. The results of a survey given to a random sample of 2,350 vaccine-eligible people indicated that 978 of the 2,350 people had received flu vaccine.
- (a) Construct a 99 percent confidence interval for the proportion of vaccine-eligible people who had received flu vaccine. Use your confidence interval to comment on the belief that 45 percent of the vaccine-eligible people had received flu vaccine.
- (b) Suppose a similar survey will be given to vaccine-eligible people in Canada by Canadian health officials. A 99 percent confidence interval for the proportion of people who will have received flu vaccine is to be constructed. What is the smallest sample size that can be used to guarantee that the margin of error will be less than or equal to 0.02 ?

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Question 8

Qualification: AP Statistics

Areas: Confidence Intervals

Subtopics: Large Counts Condition, One Sample Z Interval For A Proportion

Paper: Part-A / Series: 2016 / Difficulty: Somewhat Challenging / Question Number: 5

5. A polling agency showed the following two statements to a random sample of 1,048 adults in the United States.

Environment statement: Protection of the environment should be given priority over economic growth.

Economy statement: Economic growth should be given priority over protection of the environment.

The order in which the statements were shown was randomly selected for each person in the sample. After reading the statements, each person was asked to choose the statement that was most consistent with his or her opinion. The results are shown in the table.

	Environment Statement	Economy Statement	No Preference
Percent of sample	58%	37%	5%

- (a) Assume the conditions for inference have been met. Construct and interpret a 95 percent confidence interval for the proportion of all adults in the United States who would have chosen the economy statement.
- (b) One of the conditions for inference that was met is that the number who chose the economy statement and the number who did not choose the economy statement are both greater than 10. Explain why it is necessary to satisfy that condition.
- (c) A suggestion was made to use a two-sample z -interval for a difference between proportions to investigate whether the difference in proportions between adults in the United States who would have chosen the environment statement and adults in the United States who would have chosen the economy statement is statistically significant. Is the two-sample z -interval for a difference between proportions an appropriate procedure to investigate the difference? Justify your answer.

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Question 9

Qualification: AP Statistics

Areas: Confidence Intervals

Subtopics: Interval Estimate, Constructing a Confidence Interval, Interpreting a Confidence Interval, One Sample Z Interval For A Proportion

Paper: Part-A / Series: 2017 / Difficulty: Somewhat Challenging / Question Number: 2

2. The manager of a local fast-food restaurant is concerned about customers who ask for a water cup when placing an order but fill the cup with a soft drink from the beverage fountain instead of filling the cup with water. The manager selected a random sample of 80 customers who asked for a water cup when placing an order and found that 23 of those customers filled the cup with a soft drink from the beverage fountain.
- (a) Construct and interpret a 95 percent confidence interval for the proportion of all customers who, having asked for a water cup when placing an order, will fill the cup with a soft drink from the beverage fountain.
- (b) The manager estimates that each customer who asks for a water cup but fills it with a soft drink costs the restaurant \$0.25. Suppose that in the month of June 3,000 customers ask for a water cup when placing an order. Use the confidence interval constructed in part (a) to give an interval estimate for the cost to the restaurant for the month of June from the customers who ask for a water cup but fill the cup with a soft drink.

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Question 10

Qualification: AP Statistics

Areas: Confidence Intervals, Sampling Methods

Subtopics: Point Estimate, Given Confidence Interval - calculate n, Bias, One Sample Z Interval For A Proportion, Expected Value

Paper: Part-A / Series: 2018 / Difficulty: Somewhat Challenging / Question Number: 2

2. An environmental science teacher at a high school with a large population of students wanted to estimate the proportion of students at the school who regularly recycle plastic bottles. The teacher selected a random sample of students at the school to survey. Each selected student went into the teacher's office, one at a time, and was asked to respond yes or no to the following question.

Do you regularly recycle plastic bottles?

Based on the responses, a 95 percent confidence interval for the proportion of all students at the school who would respond yes to the question was calculated as $(0.584, 0.816)$.

- (a) How many students were in the sample selected by the environmental science teacher?
- (b) Given the method used by the environmental science teacher to collect the responses, explain how bias might have been introduced and describe how the bias might affect the point estimate of the proportion of all students at the school who would respond yes to the question.
- (c) The statistics teacher at the high school was concerned about the potential bias in the survey. To obtain a potentially less biased estimate of the proportion, the statistics teacher used an alternate method for collecting student responses. A random sample of 300 students was selected, and each student was given the following instructions on how to respond to the question.
- In private, flip a fair coin.
 - If heads, you must respond no, regardless of whether you regularly recycle.
 - If tails, please truthfully respond yes or no.
- (i) What is the expected number of students from the sample of 300 who would be required to respond no because the coin flip resulted in heads?
- (ii) The results of the sample showed that 213 of the 300 selected students responded no. Based on the results of the sample, give a point estimate for the proportion of all students at the high school who would respond yes to the question.

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Question 11

Qualification: AP Statistics

Areas: Confidence Intervals

Subtopics: Using Interval to Justify Claim, One Sample Z Interval For A Proportion

Paper: Part-A / Series: 2022 / Difficulty: Medium / Question Number: 4

4. A survey conducted by a national research center asked a random sample of 920 teenagers in the United States how often they use a video streaming service. From the sample, 59% answered that they use a video streaming service every day.
- (a) Construct and interpret a 95% confidence interval for the proportion of all teenagers in the United States who would respond that they use a video streaming service every day.
- (b) Based on the confidence interval in part (a), do the sample data provide convincing statistical evidence that the proportion of all teenagers in the United States who would respond that they use a video streaming service every day is not 0.5? Justify your answer.

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